Subway Congestion

Formulation of NE and ideal toll rates in a toll usage scheme for congestion deterrence

Mason duBoef

Overview

- Model subways as a binary transport game with tolls set for each station
- Show efficient Nash Equilibria exists and is reachable at scale using real world data
- Calculate ideal toll rate given differing train conditions and a set of commuters with distinct preference profiles

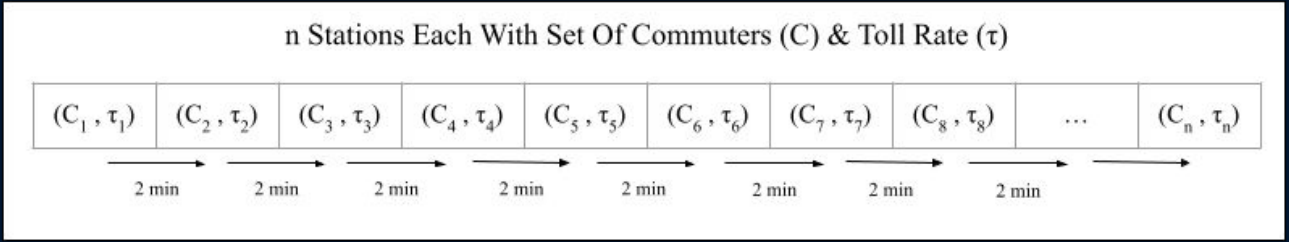
Motivation

- Setting tolls efficiently can maximize net utility during congestion
- Applies to all forms of stop-based transport
  - ie. Bus routes
- Large amount of data available from MTA

Modeling

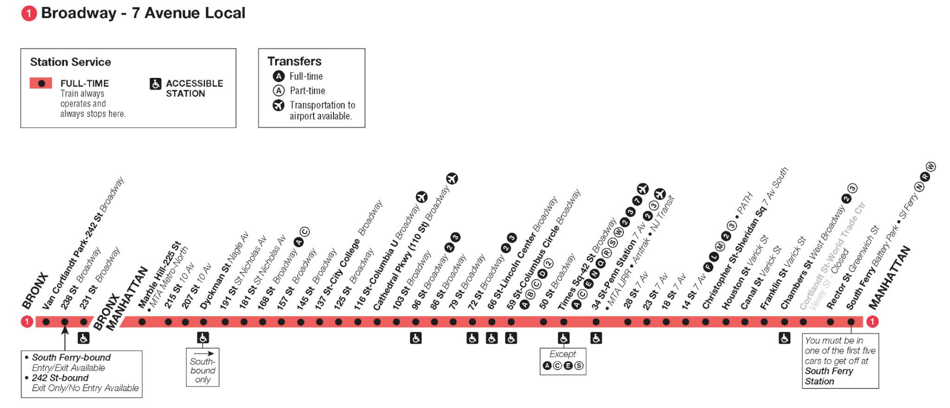
Train Modeling

- Given a set of s stations, each with set of C commuters considering entering
- Toll setting agent at each station
  - Tolls available in $0.50 increments $0-$4
- 2 minutes between neighboring stations
- Aim for 80% occupancy (arbitrary)



Simulation : NYC 1 Line

- 38 subway stations
- Number of passengers follows 2018 data with randomized variance
- 2018 time tables showing how often trains arrive
- R62A: 10 car train, 42 per car, capacity of 420
  - Ideal occupancy of 338 passengers (80%)



Commuter Modeling

V : Value of Time (Maximum amount of money willing to pay for to save 1 minute of transport)

τ_A : Toll at station S

τ_B : Monetary price of taking alternative B

T_A : Time for subway route, A

T_B : Time for alternative route, B

$$u_A = -T_A - \frac{\tau_A}{V}$$

$$u'_A = -T_A - \frac{0}{V} = T_A$$

$$u_B = -T_B - \frac{\tau_B}{V}$$

$$\delta_A = \begin{cases} 1, & \text{if } u_A \geq u_B \\ 0, & \text{otherwise.} \end{cases}$$

$$\delta_B = \begin{cases} 1, & \text{if } u_A < u_B \\ 0, & \text{otherwise.} \end{cases}$$

$$U = \delta_A * u_A + \delta_B * u_B$$

$\hat{C}$: Set of all commuters near station S

$$C = \{x \in \hat{C} | u'_A \geq u_B\}$$

Commuter Modeling : Value of Time

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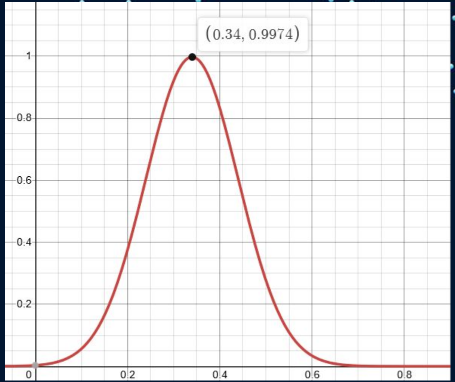

$$f(x) = \frac{1}{4 \cdot 0.1 \cdot \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-0.34}{0.1}\right)^2}$$

Median: $0.39/min VoT, $23.43/hr

Standard deviation: $0.10

Commuter Modeling : Randomized Ranges

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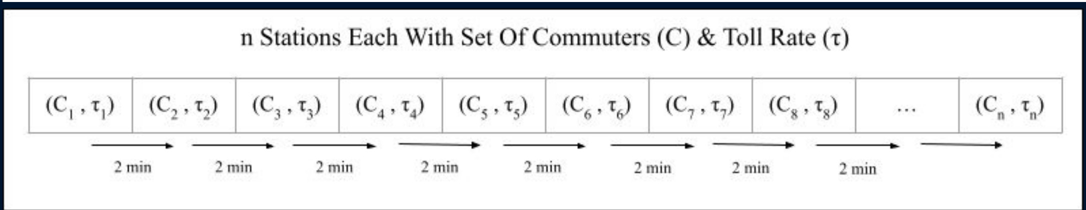

- τ_B : (\$0, \$5), \$0.50 increments
- T_A : (2, r*2), 2 min increments
- T_B : (1, r*3)

- Where r = number of remaining stations in line
- Re-randomized if $u'_A < u_b$

Ideal Toll

Formulation

Toll Formulation

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$$U = \delta_A * u_A + \delta_B * u_B$$

$$u_A = u_B$$

$$-T_A - \frac{\tau_{Amax}}{V} = u_B$$

$$\tau_{Amax} = -V(u_B + T_A)$$

Toll Formulation

c: Number of commuters in set C

Set of Commuters (C) Ordered By τ_{Amax}

$\tau_{Amax,\,1}$	$\tau_{Amax,\,2}$	$\tau_{Amax,\,3}$	$\tau_{Amax,\,4}$	$\tau_{Amax,\,5}$	…	$\tau_{Amax,\,c}$

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If capacity * 0.8 = occupancy + 4

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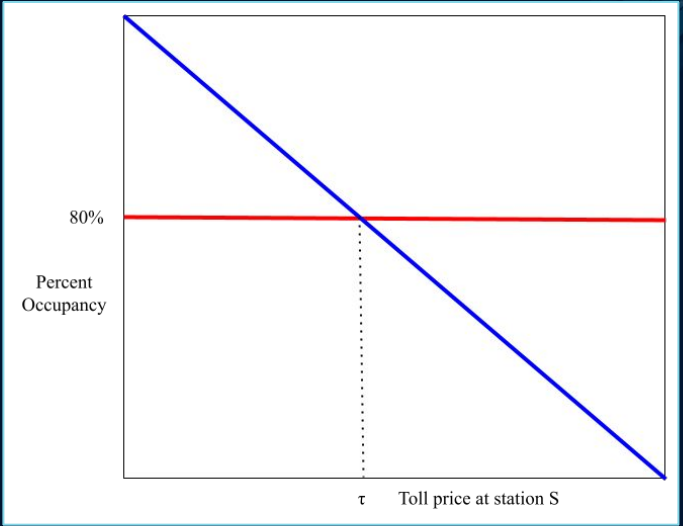

NE Formulation

NE Formulation : Myopic Best Response

- Too many possible outcomes to check every value for NE efficiently
- Define "best response"
    - NE if and only if all players are performing best response

- Commuter utility: U $U = \delta_A * u_A + \delta_B * u_B$
- Toll setter utility = -|o-(0.8*c)|

NE Formulation : Myopic Best Response

Situation: 1 short of 80% occupancy

	\$0.00	\$0.50	\$1.00	\$1.50	\$2.00	\$2.50	\$3.00	\$3.50	\$4.00
A	$(u_{A, \$0.00}, 0)$	$(u_{A, \$0.50}, 0)$	$(u_{A, \$1.00}, 0)$	$(u_{A, \$1.50}, 0)$	$(u_{A, \$2.00}, 0)$	$(u_{A, \$2.50}, 0)$	$(u_{A, \$3.00}, 0)$	$(u_{A, \$3.50}, 0)$	$(u_{A, \$4.00}, 0)$
B	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$

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B	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$

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NE Formulation : Myopic Best Response

Situation: 1 short of 80% occupancy, $\tau_{Amax} = \$2$

	\$0.00	\$0.50	\$1.00	\$1.50	\$2.00	\$2.50	\$3.00	\$3.50	\$4.00
A	$(u_{A, \$0.00}, 0)$	$(u_{A, \$0.50}, 0)$	$(u_{A, \$1.00}, 0)$	$(u_{A, \$1.50}, 0)$	$(u_{A, \$2.00}, 0)$	$(u_{A, \$2.50}, 0)$	$(u_{A, \$3.00}, 0)$	$(u_{A, \$3.50}, 0)$	$(u_{A, \$4.00}, 0)$
B	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$

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NE Formulation : Myopic Best Response

Situation: 1 short of 80% occupancy, $\tau_{Amax} = \$2$

	\$0.00	\$0.50	\$1.00	\$1.50	\$2.00	\$2.50	\$3.00	\$3.50	\$4.00
A	$(u_{A, \$0.00}, 0)$	$(u_{A, \$0.50}, 0)$	$(u_{A, \$1.00}, 0)$	$(u_{A, \$1.50}, 0)$	$(u_{A, \$2.00}, 0)$	$(u_{A, \$2.50}, 0)$	$(u_{A, \$3.00}, 0)$	$(u_{A, \$3.50}, 0)$	$(u_{A, \$4.00}, 0)$
B	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$	$(u_B, -1)$

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Weekly Progress

- Weeks 1 & 2: Read existing transport papers & decide project goal
- Week 3: Define game space & agent strategy
    - Forumate NE on small scale / simplified scale
- Week 4: Gather real world data, refine train & commuter modeling
- **Week 5: Identify large scale ideal toll rate formulation**
- **Week 6: Identify large scale PSNE formulation**
    - **Using adapted version of MBR algorithm in Lahlou & Wynter (2017)**

Thank you!

Questions? Comments?

Non-API Data Sources

- http://web.mta.info/nyct/facts/ridership/ridership_sub.htm
- http://web.mta.info/nyct/service/pdf/t1cur.pdf
- https://data.ny.gov/Transportation/MTA-Daily-Ridership-Data-Beginning-2020/vxuj-8kew
- https://www.bls.gov/regions/new-york-new-jersey/news-release/occupationalemploymentandwages_newyork.htm#tablea.f.1
- https://www.bls.gov/oes/current/oes_ny.htm